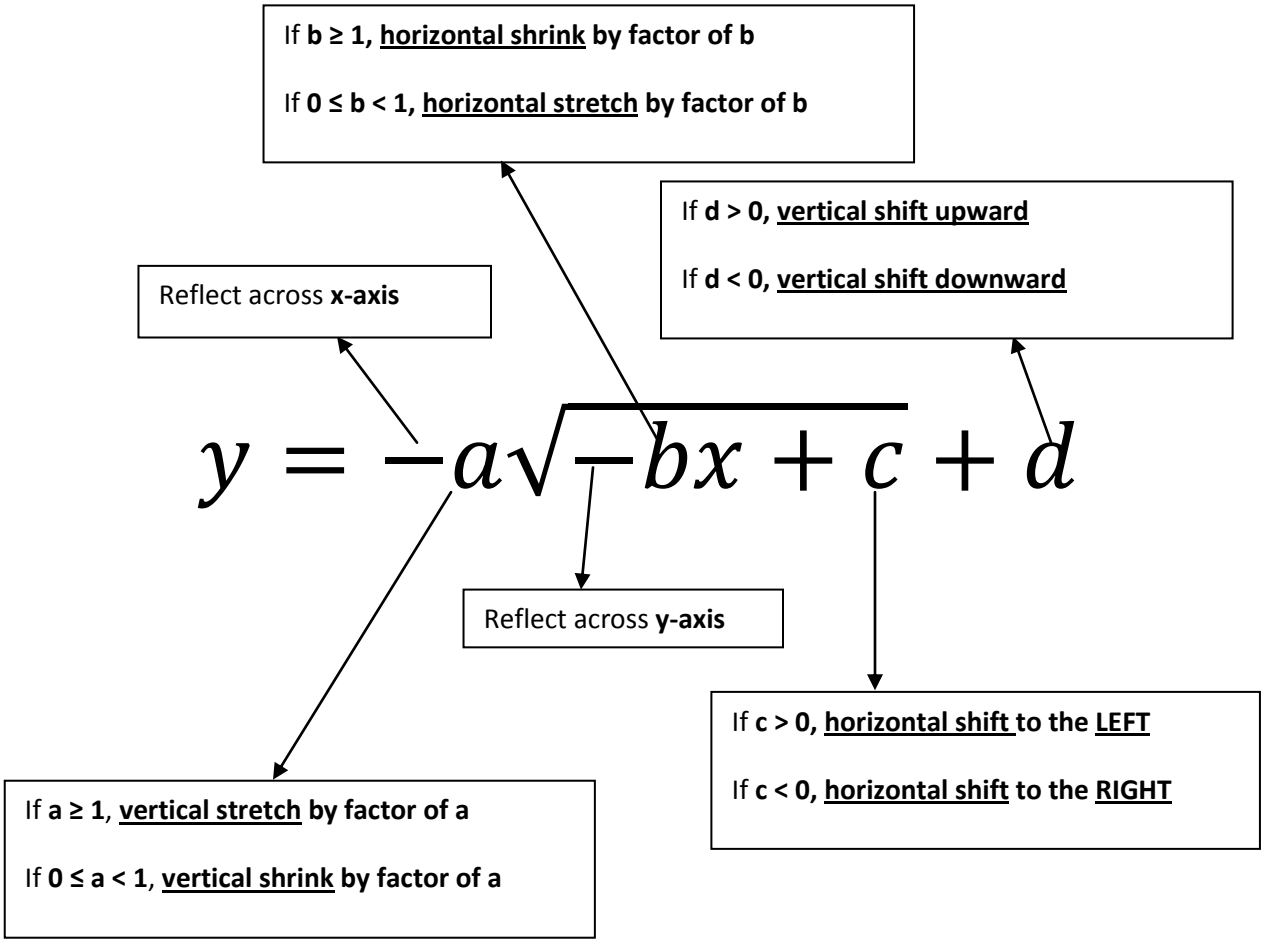
Transformations of Functions

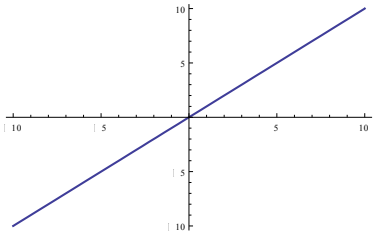
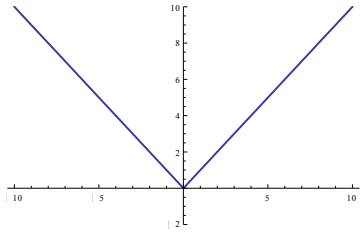
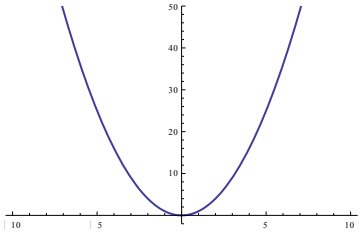
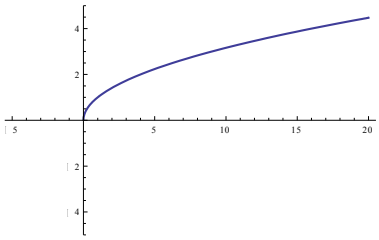
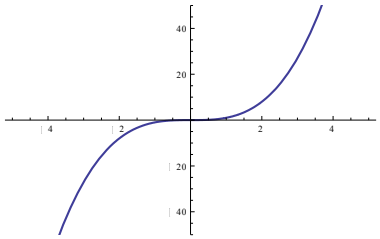
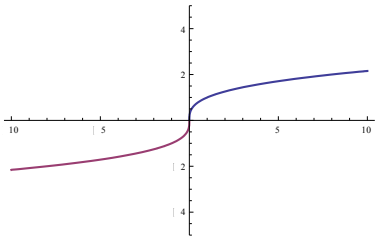
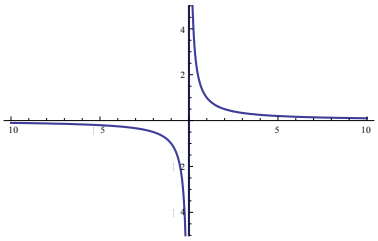
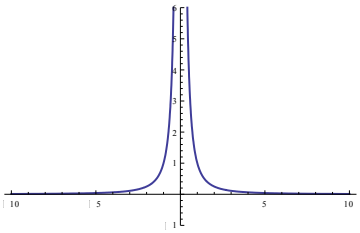
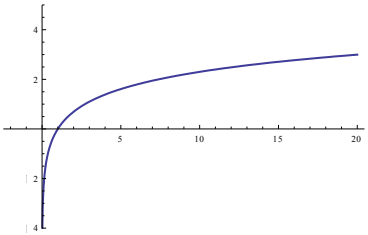
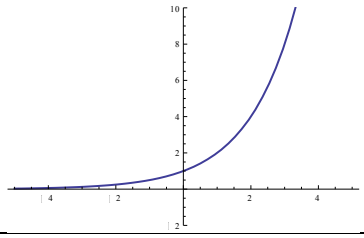
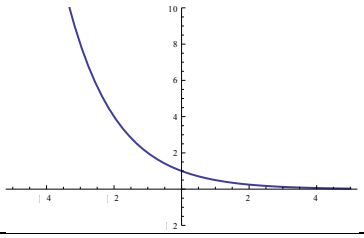


Multiple Transformations

When graphing a function that contains more than one transformation, perform the transformations in the following order:

1. Horizontal shifting
2. Stretching or Shrinking
3. Reflection
4. Vertical shifting

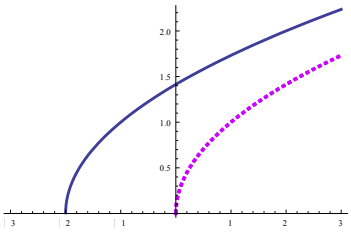
Graphs of Basic Functions

Linear Function $y = x$ 	Absolute Function $y = \|x\|$ 	Quadratic Function $y = x^2$ 
Square Root Function $y = \sqrt{x}$ 	Cube Function $y = x^3$ 	Cube Root Function $y = \sqrt[3]{x}$ 
Rational Function (odd exponent) $y = \frac{1}{x}$ 	Rational function (even exponent) $y = \frac{1}{x^2}$ 	Log Function $y = \log x$ and $x > 0$ 
Exponential Function $y = a^x$ if $a > 1$ 	Exponential Function $y = a^x$ if $0 < a < 1$ 	

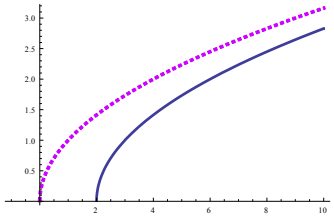
Examples of Transformations

NOTE: Dotted line is original function

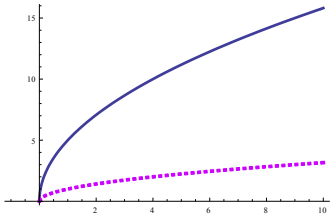
Example: Transformation of $y = \sqrt{x}$



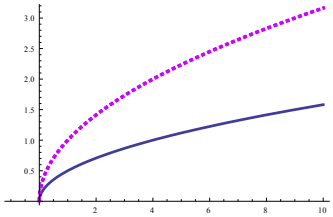
$y = \sqrt{x+2}$
Horizontal shift to the left 2 units



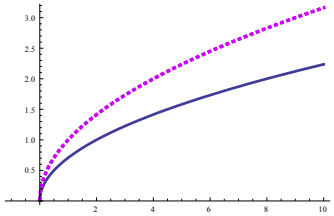
$y = \sqrt{x-2}$
Horizontal shift to the right 2 units



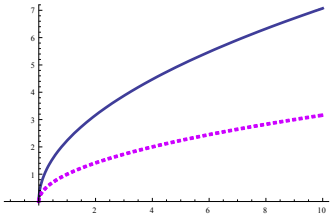
$y = 5\sqrt{x}$
Vertical stretch by factor of 5



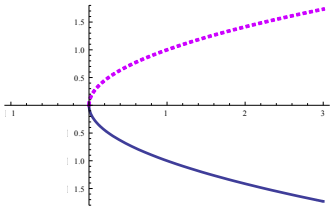
$y = 0.5\sqrt{x}$
Vertical shrink by factor of 0.5



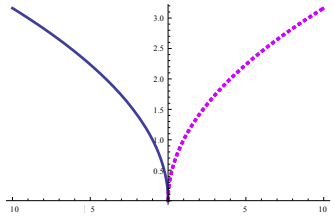
$y = \sqrt{0.5x}$
Horizontal stretch by factor of 0.5



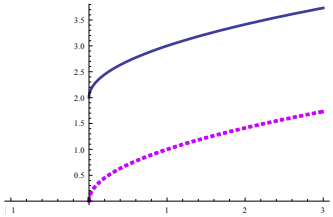
$y = \sqrt{5x}$
Horizontal shrink by factor of 5



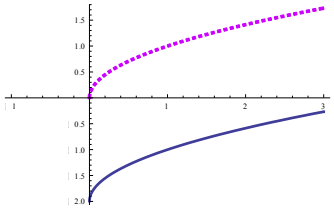
$y = -\sqrt{x}$
Reflect across x-axis



$y = \sqrt{-x}$
Reflect across y-axis



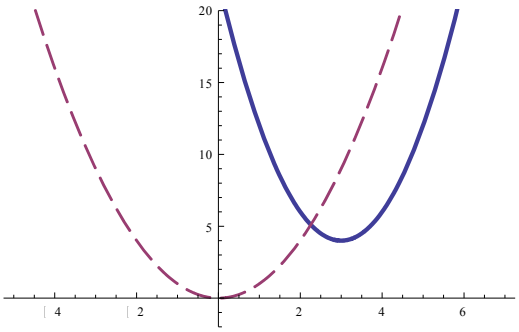
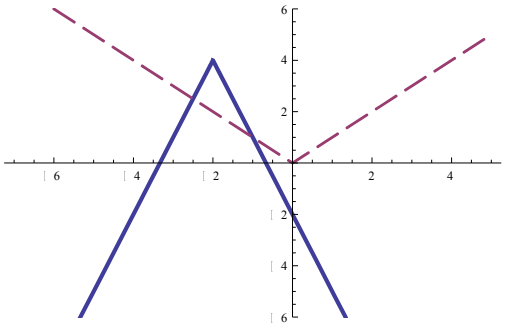
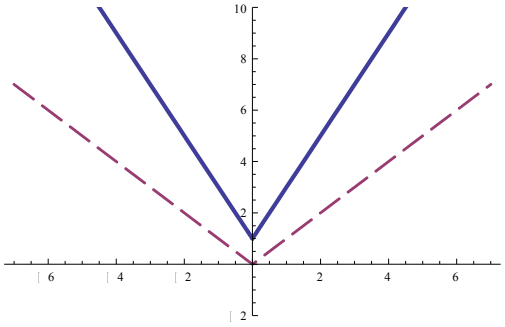
$y = \sqrt{x} + 2$
Vertical shift up 2 units



$y = \sqrt{x} - 2$
Vertical shift down 2 units

Examples of Multiple Transformations

NOTE: Dashed line is original function.

$y_1 = x^2$ $y_2 = 2(x-3)^2 + 4$ 1. Horizontal shift to the right 3 units 2. Vertical stretch by factor of 2 3. No Reflection 4. Vertical shift upward 4 units	
$y_1 = \|x\|$ $y_2 = -3\|x+2\| + 4$ 1. Horizontal shift to the left 2 units 2. Vertical stretch by factor of 3 3. Reflect across x-axis 4. Vertical shift upward 4 units	
$y_1 = \|x\|$ $y_2 = \|2x\| + 1$ 1. No Horizontal shift 2. Horizontal shrink by factor of 2 3. No Reflection 4. Vertical shift upward 1 unit	
$y_1 = x^2$ $y_2 = -(x+2)^2 + 3$ 1. Horizontal shift to the left 2 units 2. No stretch or shrink 3. Reflect across x-axis 4. Vertical shift upward 3 units